Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Assignment #5
Precedecessors
*Due on Friday, 10 October.**

This assignment is about defining and using a "step one back" *predecessor* function, $P(n)$, on the natural numbers, where the successor function, $S(n)$, is "one step forward". Informally, $P(n+1) = n$ for all n, except that we have $P(0) = 0$ because 0 is as far as we can step back in the natural numbers.

1. Give an inductive definition of $P(n)$. *[2]*

2. Prove that $P(S(n)) = n$ for all $n \in \mathbb{N}$. *[2]*

3. Show that $S(n)$ is 1–1, *i.e.* for all n, $m \in \mathbb{N}$, if $n \neq m$, then $S(n) \neq S(m)$, but that $P(n)$ is not 1–1. *[2]*

4. Give an inductive definition of the as-close-as-we-can-get-to-subtraction function on the natural numbers, $n \ominus m = \begin{cases} n - m & n \geq m \\ 0 & n \leq m \end{cases}$. *[2]*

5. Is it true that $n \ominus (m + m) = n$ for all natural numbers n and m? Prove it or give a counterexample. *[1]*

6. Is it true that $(n \ominus m) + m = n$ for all natural numbers n and m? Prove it or give a counterexample. *[1]*

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* Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can,