Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Solutions to Assignment #5
Precedecessors
Due on Friday, 10 October.

This assignment is about defining and using a "step one back" *predecessor* function, $P(n)$, on the natural numbers, where the successor function, $S(n)$, is "one step forward". Informally, $P(n+1) = n$ for all n, except that we have $P(0) = 0$ because 0 is as far as we can step back in the natural numbers.

1. Give an inductive definition of $P(n)$. *[2]*

SOLUTION. Here it is:

- $P(0) = 0$
- Given that $P(n)$ has been defined, $P(n+1) = P(S(n)) = n$.

And that's it! ■

2. Prove that $P(S(n)) = n$ for all $n \in \mathbb{N}$. *[2]*

SOLUTION. By the definition of P – see the second part of said definition ... ■

3. Show that $S(n)$ is 1–1, *i.e.* for all n, $m \in \mathbb{N}$, if $n \neq m$, then $S(n) \neq S(m)$, but that $P(n)$ is not 1–1. *[2]*

SOLUTION. We'll show that $S(n)$ is 1–1 by proving the contrapositive of for all n, $m \in \mathbb{N}$, if $n \neq m$, then $S(n) \neq S(m)$, namely that for all n, $m \in \mathbb{N}$, if $S(n) = S(m)$, then $n = m$. Suppose, then, that $S(n) = S(m)$ for n, $m \in \mathbb{N}$. By **2** above, it follows that $n = P(S(n)) = P(S(m)) = m$. Thus S is 1–1.

On the other hand, $P(n)$ is not 1–1 because $P(0) = 0 = P(S(0)) = P(1)$ and $0 \neq S(0) = 1$. Mind you, this is the only time $P(n)$ fails to be 1–1: for n, $m \in \mathbb{N} \setminus \{0\}$, if $n \neq m$, then $P(n) \neq P(m)$. ■

4. Give an inductive definition of the as-close-as-we-can-get-to-subtraction function on the natural numbers, $n \ominus m = \begin{cases} n - m & n \geq m \\ 0 & n \leq m \end{cases}$. *[2]*

SOLUTION. Here we go:

- For all $n \in \mathbb{N}$, $n \ominus 0 = n$.
- For all $n \in \mathbb{N}$, given that $n \ominus m$ has been defined, $n \ominus S(m) = P(n \ominus m)$.

And that is it! ■

5. Is it true that $n \ominus (m + m) = n$ for all natural numbers n and m? Prove it or give a counterexample. *[1]*

SOLUTION. Here is a counterexample: $1 \ominus (2 + 2) = 1 \ominus 4 = 0 \neq 1$. ■

6. Is it true that $(n \ominus m) + m = n$ for all natural numbers n and m? Prove it or give a counterexample. *[1]*

SOLUTION. Here is a counterexample: $(1 \ominus 2) + 2 = 0 + 2 = 2 \neq 1$. ■