

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2025

Solution to Assignment #2

Knights and Knaves

Due on Friday, 19 September.*

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet nine inhabitants: Bozo, Zoey, Marge, Bart, Zed, Joe, Dave, Sue and Mel. Bozo claims that it's false that Dave is a knave. Zoey tells you that Marge is a knave and Sue is a knight. Marge claims that Bozo is a knave. Bart claims, "Zed could claim that I am a knight." Zed says that only a knave would say that Dave is a knave. Joe says, "It's not the case that Bart is a knave." Dave claims that Mel and Joe are both knights or both knaves. Sue tells you that Zed is a knave. Mel claims, "Neither Bozo nor Zoey are knaves."

1. Determine, as best you can, who is a knight and who is a knave. Please give your complete reasoning. [10]

SOLUTION. (*In lots of words.*) Assume first that Bozo is a knave, and hence lies all the time. Let's see what we can deduce from this assumption.

1. Since Bozo claims that it's false that Dave is a knave, it must actually be the case that Dave is a knave.
2. Zed says that only a knave would say that Dave is a knave. Since Dave is a knave, saying so is something only a knight would say, so Zed must be a knave.
3. Since Zed is a knave and always lies, the statement that Zed could claim that Bart is a knight is true only if Bart is a knave, and is false only if Bart is a knight. This means that Bart can tell the truth by saying "Zed could claim that I am a knight" only if he is a liar, and can be telling a lie by saying "Zed could claim that I am a knight" only if he is a truth-teller. This is a contradiction ...

Since it leads to a contradiction, the assumption that Bozo is knave must be incorrect. Thus Bozo must be a knight. Let's see what we can deduce from this fact.

- a. Since Marge claims that Bozo is a knave, which is not the case, Marge is a knave.
- b. Since Bozo, who is a knight, says that it is false that Dave is a knave, Dave must be a knight.
- c. Since Zed says that only a knave would say that Dave is a knave, which is true since Dave is a knight, Zed must also be a knight.
- d. Since Sue says that Zed is a knave, which is not true, it follows that Sue is a knave.
- e. Zoey's assertion that Marge is a knave and Sue is a knight is not true because while Marge is indeed a knave, Sue is not a knight, and so Zoey must be a knave.
- f. Mel's claim that neither Bozo nor Zoey are knaves is false because while Bozo is a knight, Zoey is knave, and so Mel is a knave.

* Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can,

- g. Since Dave, who is a knight, says that Mel and Joe are both knights or both knaves, this must be true, and as we have just deduced that Mel is a knave, Joe must also be a knave.
- h. Since Joe is a knave, his assertion that it is not the case that Bart is a knave must be false, and hence Bart must be a knave.

Thus Bozo, Dave, and Zed are knights, while Marge, Sue, Zoey, Mel, Joe, and Bart are knaves. ■

NOTE. Consider Bart's statement that "Zed could claim that I am a knight." If Bart were a knight, this statement would be true because Zed is a knight, but if Bart were a knave, this statement would be false because Zed could only say that Bart is a knave. Thus Bart's statement, while consistent with our conclusions, is useless when it comes to determining whether Bart is a knight or knave.