Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Assignment #11
Real Addition
Due on Friday, 28 November.[*]

Recall that we defined the real numbers to be the *schnitts* or *Dedekind cuts*. A schnitt is a subset $r \subseteq \mathbb{Q}$ satisfying

i. $r \neq \emptyset$ and $r \neq \mathbb{Q}$;
ii. if $p \in r$ and $q < p$ for some $q \in \mathbb{Q}$, then $q \in r$ (r is "downward closed");
iii. if $p \in r$, then there is some $q \in r$ with $p < q$ (r has no largest element).

Officially, $\mathbb{R} = \{ r \subset \mathbb{Q} \mid r \text{ is a schnitt} \}$. We then proceeded to define addition on $\mathbb{R}$ by:

$$r +_{\mathbb{R}} s = \left\{ p +_{\mathbb{Q}} q \mid p \in r \text{ and } q \in s \right\}$$

We verified in class that this is a schnitt, too.

1. Verify that the addition of real numbers (*i.e.* schnitts) is associative and commutative. *[5]*

NOTE. You may assume that the rational numbers and addition of rational numbers have all the properties you need.

We also defined $0_{\mathbb{R}} = \left\{ p \in \mathbb{Q} \mid p < 0_{\mathbb{Q}} \right\}$.

2. Show that for every $r \in \mathbb{R}$, $r +_{\mathbb{R}} 0_{\mathbb{R}} = r$. *[5]*

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.