Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Assignment #10
Real Cardinalities
Due on Friday, 21 November.[*]

Recall that two sets A and B have equal cardinality, written as $\|A\| = \|B\|$, if there is a 1–1 onto function $f : A \to B$. If all we have is that there is a 1–1 function $g : A \to B$, then $\|A\| \leq \|B\|$; the Schröder-Bernstein Theorem, which we did in class recently, tells us that $\|A\| \leq \|B\|$ and $\|B\| \leq \|A\|$ together imply that $\|A\| = \|B\|$.

1. Without using the Schröder-Bernstein Theorem, show that $\|\mathbb{R}\| = \|\mathbb{R} \times \mathbb{R}\|$. *[10]*

NOTE. You may assume that the real numbers and the basic operations on them have been defined and have the usual properties. Just in case, recall that $\mathbb{R} \times \mathbb{R} = \{ (a, b) \mid a, b \in \mathbb{R} \}$.

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.