Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Assignment #1
Imaginary Matrices
*Due on Friday, 12 September.**

Before starting on this assignment, please read through the handout *Polyas Problem Solving Principles* and keep it in mind when working through problems **1**–**3**.

Recall that the complex numbers are basically the real numbers with a square root for -1, usually denoted by i, thrown in and then closed up under the usual arithmetic operations of addition and multiplication. A little more formally, the set of complex numbers is $\mathbb{C} = \{\, a + bi \mid a, b \in \mathbb{R} \,\}$, with $+$ and $\cdot$ defined by v$(a + bi) + (c + di) = (a + c) + (b + d)i$ and $(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$. Note that his definition of multiplication gives us $i^2 = (0 + 1i)^2 = -1 + 0i = -1$. We also have that $\mathbb{R} = \{\, a + bi \in \mathbb{C} \mid b = 0 \,\}$ is a subset of $\mathbb{C}$.

Let $\mathbf{M}_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \; a, b, c, d \in \mathbb{R} \right\}$ be the set of 2×2 matrices with entries from the real numbers, and let $\mathbf{O}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ be the 2×2 zero and identity matrices, respectively, in $\mathbf{M}_2(\mathbb{R})$.

1. Find a matrix $\mathbf{T} \in \mathbf{M}_2(\mathbb{R})$ such that $\mathbf{T}^2 = -\mathbf{I}_2$. *[2]*

2. Explain why there is a copy of the complex numbers in $\mathbf{M}_2(\mathbb{R})$, with this copy using the addition and multiplication of matrices as its addition and multiplication. *[3]*

The next step beyond the complex numbers are the *quaternions*, usually denoted by $\mathbb{H}$. They were invented/discovered in 1843 by William Rowan Hamilton (1805-1865), who used them to do things we mostly do with cross-products nowadays. To make the quaternions, you throw three different square roots of -1 – usually denoted by i, j, and k – into the real numbers which have a non-commutative multiplication among themselves. To be precise, we have:

$$\begin{array}{ccc} i^2 = -1 & j^2 = -1 & k^2 = -1 \\ ij = k & jk = i & ki = j \\ ji = -k & kj = -i & ik = -j \end{array}$$

Let $\mathbf{M}_4(\mathbb{R})$ be the set of 4×4 matrices with entries from thereal numbers, and let $\mathbf{O}_4$ and $\mathbf{I}_4$ be the 4×4 zero and identity matrices, respectively.

3. Find matrices $\mathbf{U}$, $\mathbf{V}$, $\mathbf{W} \in \mathbf{M}_4(\mathbb{R})$ such that:

$$\begin{array}{ccc} \mathbf{U}^2 = -\mathbf{I}_4 & \mathbf{V}^2 = -\mathbf{I}_4 & \mathbf{W}^2 = -\mathbf{I}_4 \\ \mathbf{UV} = \mathbf{W} & \mathbf{VW} = \mathbf{U} & \mathbf{WU} = \mathbf{V} \\ \mathbf{VU} = -\mathbf{W} & \mathbf{WV} = -\mathbf{U} & \mathbf{UW} = -\mathbf{V} \end{array} \quad \textit{[3]}$$

One could go on to use these matrices to show that there is a copy of the quaternions in $\mathbf{M}_4(\mathbb{R})$, but we'll save that as a possibility for another day. :-)

4. To what extent did your process in solving questions **1**–**3** follow the advice given in *Polyas Problem Solving Principles*? *[2]*

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* Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can,