Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2024

Assignment #9
The Real Numbers
Due on Friday, 15 November.[*]

Recall that we defined a *schnitt* or *Dedekind cut* to be a set S of rational numbers satisfying the following conditions:

1. $S \subset \mathbb{Q}$ and $S \neq \mathbb{Q}$.
2. S is "downward closed": if $s \in S$, $q \in \mathbb{Q}$, and $q < s$, then $q \in S$.
3. S has no largest element: if $s \in S$, there is a $t \in S$ such that $s < t$.

Intuitively, each schnitt corresponds to the real number that is the least upper bound of the schnitt, *i.e.* each real number r corresponds to the schnitt $\{\, q \in \mathbb{Q} \mid q < r \,\}$.

Officially, the real numbers *are* the schnitts, so $\mathbb{R} = \{\, S \subset \mathbb{Q} \mid S \text{ is a schnitt} \,\}$. We proceeded to define the linear order on $\mathbb{R}$ by

- $S <_{\mathbb{R}} T$ if and only if $S \subsetneq T$,

addition by

- $S +_{\mathbb{R}} T = \{\, s + t \mid s \in S \text{ and } t \in T \,\}$,

and multiplication by

- [BETTER LEFT UNWRITTEN AND FORGOTTEN].

1. Verify that $<_{\mathbb{R}}$ is indeed a linear order. *[5]*

2. Show that if S is any schnitt, then $S +_{\mathbb{R}} 0_{\mathbb{R}} = S$, where $0_{\mathbb{R}} = \left\{\, q \in \mathbb{Q} \mid q < 0_{\mathbb{Q}} \,\right\}$. (You need not show that $0_{\mathbb{R}}$ is a schnitt.) *[5]*

Algorhyme

I think that I shall never see
a graph more lovely than a tree.
A tree whose crucial property
is loop-free connectivity.
A tree that must be sure to span
so packet can reach every LAN.
First, the root must be selected.
By ID, it is elected.
Least-cost paths from root are traced.
In the tree, these paths are placed.
A mesh is made by folks like me,
then bridges find a spanning tree.

Radia Perlman

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.