Mathematics 2200H – Mathematical Reasoning TRENT UNIVERSITY, Fall 2024

Assignment #7 Omnia Gallia in partes tres divisa est[†] Due on Friday, 1 November.*

Recall that if a and b are natural numbers (or, more generally, integers), then a|b (read as "a divides b" or "a is a divisor of b or "a is a factor of b") means that there is a natural number (or integer) c such that ac = b. Also, d is the greatest common divisor of a and b, often written as gcd(a, b) or even just (a, b), if d is a divisor of both a and b and if c any other divisor of both a and b, then c|d. The Euclidean algorithm, which we looked at in class, guarantees that any two positive natural numbers do have a greatest common divisor. A corollary to this is that if d = gcd(a, b), then there exist integers – not necessarily natural numbers – x and y such that ax + by = d. Recall also that a natural number p > 1 is said to be a prime number if p has no divisors other than itself and 1. In what follows, a, b, c, and n are positive natural numbers.

- 1. Show that if c|ab and gcd(a, c) = 1, then c|b. [3]
- **2.** Show that if a|c, b|c, and gcd(a, b) = 1, then ab|c. [3.5]
- **3.** Show that the prime power decomposition of n > 1 is unique. [3.5]

Let us now praise prime numbers With our fathers that begat us: The power, the peculiar glory of prime numbers Is that nothing begat them, No ancestors, no factors, Adams among the multiplied genartions. Non can foretell their coming. Among the ordinal numbers They do not reserve their seats, arrive unexpected. Along the line of cardinals They rise like surprising pontiffs, Each absolute, inscrutable, self-elected. In the beginning where chaos Ends and zero resolves, They crowd the foreground prodigal as forest, But the middle distance thins them, Far distance to infinity Yields them as rarely returning comets. O prime improbable numbers, Long may formula-hunters Steam in abstraction, waste to skeleton patience: Stay non-conformist, nuisance, Phenomena irreducible To system, sequence, pattern or explanation.

By Helen Spalding, appearing in the December 1980 issue of Scientific American.

[†] "All Gaul is dicided into three parts," is the opening of Julius Caesar's account of his conquest of Gaul, *De Bello Gallico* ("The Gallic War").

^{*} Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.