Mathematics 2200H – Mathematical Reasoning TRENT UNIVERSITY, Fall 2024

Assignment #4 A Bit of Algebra via Set Theory and Induction Due on Friday, 4 October.*

- **1.** Use the Zermelo-Fraenkel axioms of set theory to show that the successor function, $S(x) = x \cup \{x\}$ is 1–1, *i.e.* that if S(x) = S(y), then x = y. [4]
- **2.** Use induction to show that addition on the natural numbers satisfies the *Right Cancellation Law*: for all $a, b, n \in \mathbb{N}$, if a + n = b + n, then a = b. [5]

Hint: $\mathbf{1}$ is the key to $\mathbf{2}$.

3. What can you deduce from the Right Cancellation Law for addition if you also know that addition on the natural numbers is commutative? [1]

Yet another logic limerick:

The Completeness of logics is Gödel's. 'Tis advice for looking for mödels: They're always existent For statements consistent, Most helpful for logical labörs.

^{*} Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.