Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2024

Assignment #11
Open or Closed?
Due on Friday, 29 November.[*]

Here are two definitions of *open* and *closed* for subsets of $\mathbb{R}$:

Definition 1. $A \subseteq \mathbb{R}$ us said to be *open* if for every $a \in A$ there is an $\varepsilon > 0$ such that $(a - \varepsilon, a + \varepsilon) \subseteq A$. $B \subseteq \mathbb{R}$ is said to be *closed* if $\mathbb{R} \setminus B$ is open.

Definition 2. $B \subseteq \mathbb{R}$ is *closed* if for every sequence $\{b_n\}$ such that each $b_n \in B$ that has a limit $\lim_{n \to \infty} b_n = b$, we also have $b \in B$. $A \subseteq \mathbb{R}$ is said to be *open* if $\mathbb{R} \setminus A$ is closed.

The first definition is more common, with the second then becoming a theorem that usually gets proved fairly early because of its utility.

1. Show that the two definitions of open and closed for subsets of $\mathbb{R}$ are equivalent. *[10]*

Note. Not every subset of $\mathbb{R}$ is open or closed. For a cheap example, an interval such as $(0, 1]$ which contains one of its endpoints, but not the other, is neither open nor closed.

Scumbag topologist "opens" a store.

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.