## Mathematics 2200H – Mathematical Reasoning TRENT UNIVERSITY, Fall 2024

## Assignment #10 Monotonic Subsequences Due on Friday, 22 November.\*

Recall from class that a sequence  $\{a_n\}$  of real numbers is said to be *monotonic* if it is nonincreasing, *i.e.*  $a_n \ge a_{n+1}$  for all n, or non-decreasing, *i.e.*  $a_n \le a_{n+1}$  for all n. A subsequence  $\{a_{n_i}\}$  of a sequence  $\{a_n\}$  is the sequence whose elements are the elements of the sequence  $\{a_n\}$ selected by a strictly increasing sequence of indices  $n_i$ . That is, if  $0 \le n_0 < n_1 < n_2 < \cdots$ , the subsequence is  $a_{n_0}, a_{n_1}, a_{n_2}, \ldots$ . Note that going from  $n_i$  to  $n_{i+1}$  could skip over a lot of natural numbers.

1. Prove that every sequence of real numbers has a monotonic subsequence. [10]

## Unified Field Theory

In the beginning there was Aristotle, And objects at rest tended to remain at rest, And objects in motion tended to come to rest, And soon everything was at rest, And God saw that it was boring. Then God created Newton, And objects at rest tended to remain at rest, But objects in motion tended to remain in motion, And energy was conserved and momentum was conserved and matter was conserved. And God saw that it was conservative. Then God created Einstein, And everything was relative, And fast things became short, And straight things became curved, And the universe was filled with inertial frames. And God saw that it was relatively general, but some of it was especially relative. Then God created Bohr, And there was principle, And the principle was quantum, And all things were quantified, But some things were still relative, And God saw that it was confusing. Then God was going to create Furgeson, And Furgeson would have unified, And he would have fielded a theory, And all would have been one, But it was the seventh day, And God rested, And objects at rest tend to remain at rest.

Tim Joseph [ANALOG, December 1975]

<sup>\*</sup> Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.