Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2023

Assignment #7
Greatest Common Divisors As Linear Combinations?
Due on Friday, 3 November.[*]

Recall that the *greatest common divisor* of two positive integers a and b is $d = \gcd(a, b)$, often abbreviated to just (a, b) by number theorists, such that $d|a$ and $d|b$ (*i.e.* d is a divisor of both a and b) and d is the largest integer that divides both a and b. Before you tackle this assignment you should probably review the Euclidean algorithm for finding the greatest common divisor of two positive integers.

1. Show that if a and b are positive integers and $d = \gcd(a, b)$, then there exist integers x and y – not necessarily positive! – such that $d = ax + by$. *[7]*

Hint: Run through the calculations in the Euclidean algorithm backwards ...

2. Use **1** to show that if a and b are positive integers, $d = \gcd(a, b)$, and c is a common divisor of a and b (*i.e.* $c|a$ and $c|b$), then $c|d$. *[3]*

[*] Please submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission on Blackboard fails, please submit your solutions to the instructor on paper or via email to `sbilaniuk@ trentu.ca` as soon as you can.