Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2023

Assignment #6 – *typo corrected 2023-10-13*
Multiplication
Due on Friday, 20 October.[*]

Recall from class that having defined addition on the natural numbers, we can define multiplication on the natural numbers in a similar way:

$n \cdot 0 = 0$ for all $n \in \mathbb{N}$.
For all $n \in \mathbb{N}$, given that $n \cdot k$ has been defined for some $k \in \mathbb{N}$, $n \cdot S(k) = (n \cdot k) + n$.

Very roughly, if you think of multiplication as repeated addition, then multiplication is to addition as addition is to the successor function.

In what follows, you may use the Peano Axioms, mathematical induction, and the definition and properties of addition on the natural numbers done in class.

1. Show that $n \cdot 1 = n \cdot S(0) = n$ for all $n \in \mathbb{N}$. *[1]*

2. Prove the *Left Distributive Law*: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ for all a, b, $c \in \mathbb{N}$. *[3]*

3. Show that multiplication is *associative* $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all a, b, $c \in \mathbb{N}$. *[3]*

Hint: Showing that addition is associative is similar to what you need to do here in many ways.

4. Show that multiplication is *commutative* $a \cdot b = b \cdot a$ for all a, $b, \in \mathbb{N}$. *[3]*

Hint: Showing that addition is commutative is similar to what you need to do here in many ways.

[*] Please submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission on Blackboard fails, please submit your solutions to the instructor on paper or via email to `sbilaniuk@ trentu.ca` as soon as you can.