Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2023
Assignment #10
Complex Numbers
Due on Friday, 24 November.[*]

Once the real numbers are defined, the next step is to extend them to the *complex numbers*, usually denoted by $\mathbb{C}$, by adding in a square root for -1, usually denoted by i. Informally, we then have $\mathbb{C} = \{\, a + bi \mid a,\, b \in \mathbb{R} \,\}$. Addition and multiplication of complex numbers obey all the usual rules, with the twist that $i^2 = -1$, so $(a + bi) + (c + di) = (a + c) + (b + d)i$ and $(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$.

i. Give a formal definition of the complex numbers, and also of addition and multiplication of complex numbers. *[10]*

Note. Obviously, this needs to be something different from, but effectively equivalent to, the informal description given above. We built up the integers from the natural numbers, the rational numbers from the integers, and the real numbers from the rational numbers, in various "official" formal ways that didn't necessarily have all that much of a link with our intuition about these number systems. Take inspiration from these!

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[*] Please submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission on Blackboard fails, please submit your solutions to the instructor on paper or via email to `sbilaniuk@ trentu.ca` as soon as you can.