**Assignment #1**
**The Monkey and the Coconuts**
*Due on Friday, 15 September.*\*

This assignment is mostly about practicing problem solving and does not require a lot of background knowledge. Please read the handout *Polya's Problem Solving Principles* before tackling this assignment.

Most of the following problem is adapted from a puzzle that appears in a short story by Ben Ames Williams, as reproduced in an article about the problem by Martin Gardner. This adaptation returns the problem to an old one that, according to Gardner, Williams adapted from some source that Gardner doesn't give.

> Five men and a monkey were shipwrecked on a desert island, and they spent the first day gathering coconuts for food. They piled them all up together and then went to sleep for the night.
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> But when they were all asleep one man woke up, and he thought there might be a row about dividing the coconuts in the morning, so he decided to take his share. So he divided the coconuts into five piles. He had one coconut left over, and he gave that to the monkey, and he hid his pile and put the rest all back together.
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> By and by the next man woke up and did the same thing. And he had one left over, and he gave it to the monkey. And all five of the men did the same thing, one after the other; each one taking a fifth of the coconuts in the pile when he woke up, and each one having one left over for the monkey. And in the morning they divided what coconuts were left, and they came out in five equal shares plus one coconut left over that they gave to the monkey. Of course each one must have known there were coconuts missing; but each one was as guilty as the others, so they didn't say anything. How many coconuts were there in the beginning?

**0.** How many coconuts does the monkey get in this process? *[0.5]*

**1.** Explain how the problem can be reduced to solving the equation $1024n = 15625k + 11529$ for integers $n$ and $k$, where $n$ is the number of coconuts in the beginning, and $k$ is the number of coconuts each sailor received in the final division. *[2.5]*

**2.** Explain why iit is that if $n$ is a solution of this equation for some $k$, then $n + 5^6$ is a solution of this equation for some different $k$. *[2]*

**3.** It is hard to find positive integer solutions to the equation $1024n = 15625k + 11529$ by trial and error, but it's fairly easy to find one with a negative $n$. Do so, and use this to find the smallest positive solution. *[3]*

**4.** Consider the solution you worked out to the given problem in the course of doing question **1**–**3**. How does it fit (or not!) with the general methods proposed by Polya? *[2]*

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