

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2022

Solution to Assignment #1

A Tale of the Island of Knights and Knaves

There is a class of logic puzzles, popularized by the philosopher and logician Raymond Smullyan, set on an island in which some of the inhabitants, the “knights”, always tell the truth, and the rest of the inhabitants, the “knaves”, always lie. The following is a slightly modified version of one of Smullyan’s own knights and knaves puzzles.

Once when I visited the island of knights and knaves, I came across two of the inhabitants resting under a tree. I asked one of them, “Is either of you a knight?” I received a response, and I knew the answer to my question.

Is the person to whom I addressed the question a knight or a knave? What of the other one? I can assure you that I have given you enough information to solve this problem.

1. Solve the puzzle given above. [10]

SOLUTION. Here is Smullyan’s own solution, slightly edited:

To solve this problem, you must use the information I gave you that after the speaker’s response, I knew the true answer to my question.

Suppose the speaker – call them A – had answered “Yes.” Could I have known then known whether at least one of them was a knight? Certainly not. For it could be that A was a knight and truthfully answered “Yes” (which would be truthful, since at least one – namely A – was a knight), or it could be that both of them were knaves, in which case A would have falsely answered “Yes” (which would indeed be false since neither was a knight). So if A had answered “Yes” I would have no way of knowing. But I told you that I did know after A’s answer. Therefore A must have answered “No.”

The reader can now easily see what A and the other – call them B – must be: If A were a knight, they couldn’t have truthfully answered “No,” so A is a knave. Since the answer “No” is false, there is at least one knight between the two of them. Hence A is a knave and B is a knight.

The given problem is puzzle #36 in Smullyan’s *What Is the Name of This Book?*, from which the given solution was also taken. ■