Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2022

Assignment #8 [cleaned up a bit]
What comes before!
*Due on Friday, 11 November.**

Please your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Suppose $\mathbb{Q}$ and the usual linear order on the rationals (usually denoted by $<$, or by $<_{\mathbb{Q}}$ when you have other linear orders to keep track of) are defined as they were in class:

- Define the equivalence relation $\approx$ on pairs of integers by
  $(a, b) \approx (c, d)$ if and only if $ad = bc$.
- If (a, b) is a pair of integers with $b \neq 0$, let
  $[(a, b)]_{\approx} = \{ (c, d) \in \mathbb{Z} \times \mathbb{Z} \mid (a, b) \approx (c, d) \}$
  Intuitively, this equivalence class represents the fraction $\frac{a}{b}$.
- Let $\mathbb{Q} = \left\{ [(a, b)]_{\approx} \mid a,\, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$.
- We can assume that the second coordinate in the pair defining an equivalence class is positive (intuitively, we can assume that the denominator of any fraction is positive). If we stick to such pairs as representatives of equivalence classes, then $[(a, b)]_{\approx} <_{\mathbb{Q}} [(c, d)]_{\approx}$ if and only if $ad <_{\mathbb{Z}} bc$.

In answering the questions below, you may assume that all the familiar properties of the integers, as well as the operations and linear order on the integers, are true.

1. Using these definitions of $\mathbb{Q}$ and $<_{\mathbb{Q}}$, show that $\mathbb{Q}$ has no endpoints, *i.e.* $\mathbb{Q}$ has no smallest and no largest element. *[5]*

2. Show that $\mathbb{Q}$ is *countable*, that is, that there is a 1–1 onto function $f : \mathbb{N} \to \mathbb{Q}$. *[5]*

NOTE. Any such function will not play well with the respective arithmetic operations or relations in each number system

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* You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instructor at `sbilaniuk@trentu.ca`.