Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2022

Assignment #7

What comes before!

Due on Friday, 4 November.[*]

Please your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Suppose $\mathbb{Z}$ has been successfully defined somehow or other, along with the linear order $<$ on $\mathbb{Z}$. However, we have not been given any operations on it except the successor function $S : \mathbb{Z} \to \mathbb{Z}$, which for each $a \in \mathbb{Z}$ gives us the next larger integer, the predecessor function $P : \mathbb{Z} \to \mathbb{Z}$, which for each $a \in \mathbb{Z}$ gives us the next smaller integer, and the negative function $N : \mathbb{Z} \to \mathbb{Z}$, which for each $a \in \mathbb{Z}$ gives us the integer $N(a)$ exactly as far from 0 as a, but on the other side of 0. Informally, of course, $S(a) = a + 1$, $P(a) = a - 1$, and $N(a) = -a$; this has to be informal because we don't yet have the operations of addition and subtraction.

1. Use the given functions to define $+$ and $-$ on $\mathbb{Z}$ inductively. *[7]*

2. Use your definitions of $+$ and $-$ to show that for all integers $a \in \mathbb{Z}$, $a + N(a) = 0$. *[3]*

Some Halloween Reading

A Tale of the Thirteenth Floor, by the American poet Ogden Nash. His one and only horror ballad, as far as I know.

The Ballad of the Black Fox Skin, a horror ballad by the Canadian poet Robert W. Service. One of his lesser-known works, but quite gripping.

A Night in the Lonesome October, a novel by the American fantasist Roger Zelazny. The blurb "A heartwarming tale of black magic and murder, told from the standpoint of Jack the Ripper's dog Snuff" is accurate!

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[*] You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instructor at `sbilaniuk@trentu.ca`.