

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2022

Assignment #5

Multiplication

*Due on Friday, 14 October.**

Please your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Early in the term, and again more recently, we defined multiplication on the natural numbers from addition as follows:

- Let $n \cdot 0 = 0$ for all $n \in \mathbb{N}$.
- Given that $n \cdot k$ has been defined, let $n \cdot S(k) = (n \cdot k) + n$, where S is the successor function.

1. Show that multiplication of natural numbers is associative, that is, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in \mathbb{N}$. [2]
2. Show that multiplication of natural numbers is commutative, that is, $a \cdot b = b \cdot a$ for all $a, b \in \mathbb{N}$. [3]

NOTE. For **1** and **2** the proofs done in class that addition of natural numbers is associative and commutative, respectively, are useful models for how to proceed.

3. Define exponentiation of natural numbers (with 0^0 set to 1) and prove what algebraic properties you can about it. [3]

Hint: Exponentiation is to multiplication as multiplication is to addition, more or less. However, exponentiation is neither associative nor commutative, though it has some basic algebraic properties that are useful and you should already be familiar with using.

* You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instructor at sbilaniuk@trentu.ca.