Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2022

Assignment #4
The Fibonacci Sequence and the Golden Ratio
Due on Friday, 7 October.[*]

Please your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Leonardo of Pisa (*c.* 1170 A.D. – *c.* 1250 A.D.), nowadays commonly known as Fibonacci, is usually considered to be the best European mathematician of the Middle Ages. He is most famous nowadays for a problem in his book *Liber Abaci* (1202 A.D., revised *c.* 1227 A.D.) that involved what is now called the Fibonacci sequence.[†] It is usually defined recursively as follows:

$$f_0 = 0,\ f_1 = 1, \text{ and } f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 2.$$

This sequence turns up in surprising places, such as the arrangement of seeds in a sunflower. It also has connections with the so-called *Golden Ratio*, which can be defined as the positive real number φ satisfying the equation $\varphi = \dfrac{\varphi+1}{\varphi} = 1 + \dfrac{1}{\varphi}$, which also turns up in surprising places. This assignment will have you work through a couple of bits of this connection.

1. Show that $\varphi = \dfrac{1+\sqrt{5}}{2}$. *[1]*

2. Use induction to show that $f_n = \dfrac{\varphi^n - (-\varphi)^{-n}}{\varphi + \varphi^{-1}}$ for all $n \geq 0$. *[6]*

3. Show that $\displaystyle\lim_{n \to \infty} \frac{f_{n+1}}{f_n} = \varphi$. *[3]*

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[*] You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instructor at `sbilaniuk@trentu.ca`.

[†] The sequence was already known to Indian mathematicians by *c.* 700 A.D., and possibly several centuries earlier, depending on how one interprets some ambiguous language in the relevant texts.