Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2022
Assignment #11
Reality bites?
Due on Friday, 2 December.[*]

Please give your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Recall from class (or Assignment #10) that a set S of rational numbers is a *schnitt* if

(1) $S \neq \emptyset$ and $S \neq \mathbb{Q}$,
(2) S is *downward closed*, *i.e.* whenever $s \in S$, $q \in \mathbb{Q}$, and $q <_{\mathbb{Q}} s$, it follows that $q \in S$,
and (3) S has no largest element, *i.e.* for every $s \in S$, there is a $t \in S$ such that $s <_{\mathbb{Q}} t$.

We then defined the real numbers to be the schnitts, *i.e.* officially $\mathbb{R} = \{ S \subset \mathbb{Q} \mid S \text{ is a schnitt} \}$, and proceeded to define addition of real numbers, *a.k.a.* schnitts, by $S +_{\mathbb{R}} T = \{ s + t \mid s \in S \text{ and } t \in T \}$. (Multiplication was a little harder ...)

We also defined the linear order, $<_{\mathbb{R}}$, on the schnitty[†] reals by $S <_{\mathbb{R}} T \iff S \subsetneq T$.

1. A sequence $\{s_n\}$ of real numbers is non-decreasing (respectively, non-increasing) if $s_n \leq_{\mathbb{R}} s_{n+1}$ (respectively, $s_{n+1}n \leq_{\mathbb{R}} s_n$) for all n. The *Monotone Convergence Theorem* is the following statement:

> Suppose $\{s_n\}$ is a non-decreasing (respectively, non-increasing) sequence of real numbers with an upper (respectively, lower) bound. Then $\{s_n\}$ converges.

Use the schnitt definition of the real numbers to prove the Monotone Convergene Theorem for non-decreasing sequences. *[10]*

Hint: It's pretty easy to use the sequence to define the schnitt that is the limit of the sequence.

[*] You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instructor at `sbilaniuk@trentu.ca`.

[†] Sorry - I couldn't resist! :-)