Mathematics 2200H – Mathematical Reasoning
Trent University, Fall 2022
Assignment #10
Getting real!
*Due on Friday, 25 November.**

Please give your complete reasoning in your solution. Recall that, unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solution by yourself and acknowledge all sources and help that you ended up using.

Recall from class that a set S of rational numbers is a *schnitt* if

(1) $S \neq \emptyset$ and $S \neq \mathbb{Q}$,

(2) S is *downward closed*, *i.e.* whenever $s \in S$, $q \in \mathbb{Q}$, and $q <_{\mathbb{Q}} s$, it follows that $q \in S$,

and (3) S has no largest element, *i.e.* for every $s \in S$, there is a $t \in S$ such that $s <_{\mathbb{Q}} t$.

We then defined the real numbers to be the schnitts, *i.e.* officially $\mathbb{R} = \{\, S \subset \mathbb{Q} \mid S \text{ is a schnitt}\,\}$, and proceeded to define addition of real numbers, *a.k.a.* schnitts, by $S +_{\mathbb{R}} T = \{\, s + t \mid s \in S \text{ and } t \in T \,\}$.

1. Check that $0_{\mathbb{R}} = \{\, q \in \mathbb{Q} \mid q <_{\mathbb{Q}} 0_{\mathbb{Q}} \,\}$ is indeed a schnitt. *[3]*

2. Show that if S is any schnitt, then $S +_{\mathbb{R}} 0_{\mathbb{R}} = S$. *[7]*