Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2021

Assignment #8
$\mathbb{Z}$ombie Math?
Due on Friday, 12 November.
*May be submitted on paper or via Blackboard.**

Recall from class that we defined the integers by first defining an equivalence relation $\sim$ on $\mathbb{N} \times \mathbb{N} = \{ (a, b) \mid a, b \in \mathbb{N} \}$ by $(a, b) \sim (c, d) \iff a + d = b + c$, and then let the integers be the equivalence classes of this relation, *i.e.* $\mathbb{Z} = \{ [(a, b)]_\sim \mid a, b \in \mathbb{N} \}$. We then proceeded to define taking negatives, addition, multiplication, and the linear order on $\mathbb{Z}$ by:

$$\begin{aligned}
&-[(a,b)]_\sim = [(b,a)]_\sim \\
&[(a,b)]_\sim + [(c,d)]_\sim = [(a+c, b+d)]_\sim \\
&[(a,b)]_\sim \cdot [(c,d)]_\sim = [(ac+bd, ad+bc)]_\sim \\
&[(a,b)]_\sim < [(c,d)]_\sim \iff a+d < b+c
\end{aligned}$$

We also defined the unit elements for addition and multiplication by $0 = [(0, 0)]_\sim$ and $1 = [(1, 0)]_\sim$.

In answering the following questions, you may assume that all the basic properties of the arithmetic operations and the linear order on $\mathbb{N}$ have been established.

1. Suppose p, $q \in \mathbb{Z}$. Show that:

- **a.** $(-p) + (-q) = -(p + q)$ *[1]*
- **b.** $(-p) \cdot q = p \cdot (-q) = -(p \cdot q)$ *[1.5]*
- **c.** $(-p) \cdot (-q) = p \cdot q$ *[1.5]*
- **d.** $(-p) < (-q) \iff q < p$ *[1]*

2. Suppose p, q, r, $s \in \mathbb{Z}$. Show that:

- **a.** $p = q \iff p + r = q + r$ *[1.5]*
- **b.** $p < q \iff p + r < q + r$ *[2]*
- **c.** $p < q$ and $r < s \implies p + r < q + s$ *[1.5]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`