Assignment #7
The linear order on ℕ.
Due on Friday, 5 November.
*May be submitted on paper or via Blackboard.**

Recall from Assignment #5 that the usual linear order "less than" on the natural numbers can be defined by:

$a < b$ for natural numbers a and b if and only if $a \in b$.

1. Show that for all a, $b \in \mathbb{N}$, $a < b$ if and only if $b = a + S(k)$ for some $k \in \mathbb{N}$. *[4]*

As was noted in class some time ago, a (strict) linear order on a set A, let's denote it by $\lhd$, is a binary relation satisfying the following conditions:

1. *Irreflexivity:* For all $a \in A$, it is not the case that $a \lhd a$.
2. *Transitivity:* For all a, b, $c \in A$, if $a \lhd b$ and $b \lhd c$, then $a \lhd c$.
3. *Trichotomy:* For all a, $b \in A$, exactly one of $a \lhd b$, $a = b$, or $b \lhd a$, is true.

2. Show that $<$ on ℕ is a linear order. *[6]*