

**Mathematics 2200H – Mathematical Reasoning**

TRENT UNIVERSITY, Fall 2021

**Assignment #7**

**The linear order on  $\mathbb{N}$ .**

*Due on Friday, 5 November.*

*May be submitted on paper or via Blackboard.\**

Recall from Assignment #5 that the usual linear order “less than” on the natural numbers can be defined by:

$a < b$  for natural numbers  $a$  and  $b$  if and only if  $a \in b$ .

1. Show that for all  $a, b \in \mathbb{N}$ ,  $a < b$  if and only if  $b = a + S(k)$  for some  $k \in \mathbb{N}$ . [4]

As was noted in class some time ago, a (strict) linear order on a set  $A$ , let's denote it by  $\triangleleft$ , is a binary relation satisfying the following conditions:

1. *Irreflexivity*: For all  $a \in A$ , it is not the case that  $a \triangleleft a$ .
2. *Transitivity*: For all  $a, b, c \in A$ , if  $a \triangleleft b$  and  $b \triangleleft c$ , then  $a \triangleleft c$ .
3. *Trichotomy*: For all  $a, b \in A$ , exactly one of  $a \triangleleft b$ ,  $a = b$ , or  $b \triangleleft a$ , is true.

2. Show that  $<$  on  $\mathbb{N}$  is a linear order. [6]

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\* All else failing, please email your solutions to the instructor at: [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca)