Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2021

Assignment #6
Cancellation and Distributive Laws
Due on Friday, 22 October.
*May be submitted on paper or via Blackboard.**

Recall from class that addition on the natural numbers was defined by recursion with the help of the successor function S as follows:

- For all $n \in \mathbb{N}$, $n + 0 = n$.
- For all $n \in \mathbb{N}$, if $n + k$ has been defined, then $n + S(k) = S(n + k)$.

Since $\mathbb{N}$ lacks negatives, one can't really define the usual notion of subtraction in a general way, costing us a useful tool for solving equations. The following fact lets us compensate for this, at least to some extent.

CANCELLATION LAW (for $+$ on $\mathbb{N}$): For all n, m, $k \in \mathbb{N}$, if $n + k = m + k$, then $n = m$.

1. Prove the Cancellation Law for $+$ on $\mathbb{N}$. *[5]*

Given that we have defined addition on the natural numbers, we can also define multiplication on the natural numbers by recursion as follows:

- For all $n \in \mathbb{N}$, $n \cdot 0 = 0$.
- For all $n \in \mathbb{N}$, if $n \cdot k$ has been defined, then $n \cdot S(k) = n \cdot k + n$.

Note that when evaluating the expression $n \cdot k + n$, we use the normal convention that unless operations are explicitly grouped otherwise, multiplications are performed before additions. Thus $n \cdot k + n$ means $(n \cdot k) + n$, not $n \cdot (k + n)$.

Addition and multiplication are connected a little more generally via the following rule:

DISTRIBUTIVE LAW (for $\cdot$ over $+$ on $\mathbb{N}$): For all a, b, $c \in \mathbb{N}$, $a \cdot (b + c) = a \cdot b + a \cdot c$.

2. Prove the Distributive Law for $\cdot$ over $+$ on $\mathbb{N}$. *[5]*

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* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`