Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2021

Assignment #5
Going down!

Due on Friday, 15 October.
*May be submitted on paper or via Blackboard.**

Recall from class that the natural numbers can be built up as follows from the empty set using the successor function, $S(x) = x \cup \{x\}$.

$$
\begin{aligned}
&0 = \emptyset \\
&1 = S(0) = 0 \cup \{0\} = \{0\} \\
&2 = S(1) = 1 \cup \{1\} = \{0, 1\} \\
&3 = S(2) = 2 \cup \{2\} = \{0, 1, 2\} \\
&\vdots \\
&n = S(n-1) = (n-1) \cup \{n-1\} = \{0, 1, 2, \dots, n-1\} \\
&\vdots
\end{aligned}
$$

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$ then denotes the set of all natural numbers. (Of course, we had to add the Axiom of Infinity to our collection of axioms for set theory to ensure that $\mathbb{N}$ is indeed a set.) Note that each natural number, considered as a set, is just the set of all of its predecessors. One pleasant consequence of this is that we can now define $<$ on the natural numbers very easily:

$a < b$ for natural numbers a and b if and only if $a \in b$.

Recall also the Descending Chain Condition from Assignment #3, which we rephrase here a little to apply to the natural numbers in particular:

($\downarrow$) *(Descending Chain Condition)* Every strictly decreasing sequence of natural numbers is finite.

That is, if you have a sequence of natural numbers $a_0 > a_1 > a_2 > \cdots$, then it cannot be infinite.

1. Using the definitions of the natural numbers and $<$ given above, prove the Descending Chain Condition is true of the natural numbers. *[3]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`