

## Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2020

### Solution to Assignment # $\{\emptyset, \{\emptyset\}\}$

#### Knights and Knaves

*Due on Friday, 25 September.*

“Knights and Knaves” is a class of logic puzzles popularised by the logician Raymond Smullyan, who published several books of “recreational logic” puzzles, including one with the whimsical title *What Is the Name of This Book?* The puzzle below is #330 from a collection put online by the Department of Philosophy at Hong Kong University, which can be found at: <https://philosophy.hku.hk/think/logic/knights.php>

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet eight inhabitants: Sue, Carl, Homer, Dave, Zeke, Ted, Marge and Zed. Sue tells you that both Homer is a knave and Marge is a knight. Carl claims that Ted is a knave. Homer tells you that Sue is a knight or Dave is a knave. Dave tells you, “Neither Zed nor Marge are knights.” Zeke tells you that it’s false that Marge is a knave. Ted says that Carl and Zed are both knights or both knaves. Marge claims that Sue and Zed are both knights. Zed claims that Carl could claim that Homer is a knave.

1. Determine, as best you can, who is a knight and who is a knave. [10]

SOLUTION. Here goes:

First, if Sue were a knight, then her statement that “Homer is a knave and Marge is a knight” would have to be true. Thus Homer would have to be a knave, yet Homer’s statement that “Sue is a knight or Dave is a knave” would be true because Sue is a knight. Since this is a contradiction, Sue cannot be a knight. Thus Sue is a knave.

Second, since Sue is a knave, Marge’s claim that “Sue and Zed are both knights” is false because Sue is a knave. Thus Marge is knave.

Third, since Marge is a knave, Zeke’s statement that “it’s false that Marge is a knave” is false. Thus Zeke is a knave.

Fourth, suppose Carl was a knight. Then Carl’s claim that “Ted is a knave” would be true, and so Ted’s assertion that “Carl and Zed are both knights or both knaves” would be false. Since we are supposing that Carl is a knight, it would follow that Zed must be a knave. This would make Dave’s assertion that “neither Zed nor Marge are knights” true, and so Dave would have to be a knight. This, in turn, would make Homer’s statement that “Sue is a knight or Dave is a knave” false, and so Homer would have to be a knave. This, however, would mean that Carl, as a knight, could indeed claim that Homer is a knave, making Zed’s claim that “Carl could claim that Homer is a knave” true, so Zed would have to be a knight, contradicting the earlier deduction that Zed must be a knave. Since supposing Carl to be a knight leads to a contradiction, it follows that Carl must be a knave.

Fifth, since Carl is a knave, it follows that Carl’s claim that “Ted is a knave” is false. Thus Ted is a knight.

Sixth, since Ted is a knight, his statement that “Carl and Zed are both knights or both knaves” must be true. As Carl is a knave, it follows that Zed is also a knave.

Seventh, since Zed and Marge are both knaves, Dave’s statement “Neither Zed nor Marge are knights” is true, so Dave must be a knight.

Finally, Homer’s statement that “Sue is a knight or Dave is a knave” is false because Sue is a knave and Dave is a knight, so Homer must be a knave.

Thus Sue, Carl, Homer, Zeke, Marge, and Zed are knaves, while Dave and Ted are knights.

This is a valid possible solution since the statements each person makes are true or false depending on whether the person is a knight or knave, respectively, as required:

Sue is a knave and her statement that “Homer is a knave and Marge is a knight” is false because Marge is a knave.

Carl is a knave and his claim that “Ted is a knave” is false because Ted is a knight.

Homer is a knave and his statement that “Sue is a knight or Dave is a knave” is false since Sue is a knave and Dave is a knight.

Dave is a knight and his statement that “neither Zed nor Marge are knights” is true because both Zed and Marge are knaves.

Zeke is a knave and his claim that “it’s false that Marge is a knave” is false because Marge is indeed a knave.

Ted is a knight and his statement that “Carl and Zed are both knights or both knaves” is true because both Carl and Zed are knaves.

Marge is a knave and her claim that “Sue and Zed are both knights” is false since Sue and Zed are actually both knaves.

Finally, Zed is a knave and his claim that “Carl could claim that Homer is a knave” is false because Carl is a knave and so could not make the true claim that Homer is a knave.

Since multiple solutions are possible only when some person’s type cannot be determined, the solution above is the only one. ■