Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2020

Assignment #0 + 1 + 2 + 3
Skipping a few number systems ahead ... :-)
Due on Friday, 23 October.

The *quaternions* are the number system after the complex numbers:

$$\mathbb{H} = \{\, a + bi + ci + dj \mid a,\, b,\, c,\, d \in \mathbb{R} \,\}$$

where $+$ and $\cdot$ work as usual except for the special numbers i, j, and k, which satisfy the following relations:

$$\begin{aligned} &i^2 = j^2 = k^2 = -1 \\ &ij = k \quad\quad jk = i \quad\quad ki = j \\ &ji = -k \quad kj = -i \quad ik = -j \end{aligned}$$

Note that you have multiple square roots of -1 and that multiplication is not always commutative in the quaternions.

1. Suppose $\mathbf{a} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} s \\ t \\ u \end{bmatrix}$ are two vectors in $\mathbb{R}^3$ and that $(pi + qj + rk)(si + tj + uk) = a + bi + ci + dj$. Verify that $\mathbf{a} \times \mathbf{b} = \begin{bmatrix} b \\ c \\ d \end{bmatrix}$. What does the real number a represent in terms of the vectors $\mathbf{a}$ and $\mathbf{b}$? *[5]*

2. Suppose $h = a + bi + ci + dj \in \mathbb{H}$ and $h \neq 0$. Express $h^{-1} = \dfrac{1}{h}$ as a quaternion in terms of a, b, c, and d. *[5]*