

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2020

Assignment #2² + 1

Exponentiation in \mathbb{N}

Due on Friday, 16 October.

Let's define the operation of exponentiation in the natural numbers as follows:

- For all $n \in \mathbb{N}$, let $n^0 = 1$.
- Given that n^k has been defined for some $k \in \mathbb{N}$ and all $n \in \mathbb{N}$, let $n^{S(k)} = (n^k) \cdot n$.

In answering the questions below, you may use the definitions and all the properties of $+$ and \cdot on \mathbb{N} developed in the lectures, plus the (augmented) Peano axioms we are using, plus the definition of exponentiation above.

1. Prove that $0^k = 0$ for all $k \geq 0$. [2]
2. Prove that for all $n, m, k \in \mathbb{N}$, $(n^m) \cdot (n^k) = n^{m+k}$. [4]
3. Prove that for all $n, m, k \in \mathbb{N}$, $(n^k) \cdot (m^k) = (n \cdot m)^k$. [4]