

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2020

Assignment #S(2)

A little bit of Aristotle

Due on Friday, 2 October.

The Greek philosopher Aristotle seems to have been the first to write about logic as a subject in its own right. His logic relied on a number of rules of procedures or argument forms whose uses are called “syllogisms”, such as the following:

*All humans are featherless bipeds.**

Meredith is human.

Therefore Meredith is a featherless biped.

Suppose we have two one-place relations H and F in a first-order language, whose intended meanings are “ x is human” for $H(x)$ and “ x is a featherless biped” for $F(x)$, as well as a constant symbol m representing Meredith. Then the syllogism above claims that the assumptions $\forall x (H(x) \rightarrow F(x))$ and $H(m)$ imply the conclusion that $F(m)$.

1. Give a deduction, using the axiom schema given in class and with Modus Ponens as the only rule of procedure, of $F(m)$ from the premisses $\forall x (H(x) \rightarrow F(x))$ and $H(m)$. You may assume that m is substitutable for x in the formula $(H(x) \rightarrow F(x))$. [2]

2. Translate the three sentences

Some humans are featherless bipeds.

No humans are featherless bipeds.

Some humans are not featherless bipeds.

into suitable formulas of a first-order language including the relations H and F described above. [3]

3. Together with “All humans are featherless bipeds”, the sentences in **2** reflect the four main sentence forms involving quantifiers studied in Aristotle’s logic. If there are no humans, but there are some featherless bipeds, in the universe these four sentences are talking about, which of them must be true? If, instead, there are no featherless bipeds, but there are some humans in that universe, which of the four sentences must be true? (Do explain why in each case, but you need not give formal deductions.) [3]
4. Aristotelian logic pretty much consists of propositional logic, plus just enough first-order logic to properly handle the four sentence forms given above. Give an example of a mathematical result and its proof that Aristotelian logic is *not* adequate to handle, and explain why this is so. [2]

* Aristotle apparently characterized humans as “featherless bipeds” in one of his works on natural philosophy.