

Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2019
Solution to Assignment #2
Knights and Knaves

The following is Puzzle #313 out of 382 of the knights and knaves puzzle collection at:
<https://philosophy.hku.hk/think/logic/knights.php>

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet eight inhabitants: Marge, Mel, Betty, Bob, Bill, Carl, Zeke and Alice. Marge says that it's not the case that Bob is a knave. Mel claims, "Bill is a knave." Betty tells you that Carl and Zeke are knights. Bob claims that neither Mel nor Bill are knaves. Bill says, "Only a knave would say that Carl is a knave." Carl tells you, "I know that Alice is a knave and that Zeke is a knight." Zeke tells you, "Of Carl and I, exactly one is a knight." Alice claims that Zeke and Marge are both knights.

Can you determine who is a knight and who is a knave?

1. Determine, as best you can, which of the eight are knights and which are knaves. Please explain your reasoning as fully as you can. [10]

SOLUTION. Suppose that Carl is a knight. It would follow that Carl's statement that "I know that Alice is a knave and that Zeke is a knight" is true; in particular, it would follow that Zeke is a knight. However, Zeke's statement that "Of Carl and I, exactly one is a knight" would then be false, contradicting Zeke being a knight. Since assuming that Carl is a knight led to a contradiction, it must be the case that Carl is a knave.

Since Carl is a knave, Betty's statement that Carl and Zeke are knights must be false, so Betty is a knave.

Since "Carl is a knave" is a true statement, Bill's statement that "Only a knave would say that Carl is a knave" must be false, because only knights assert true statements. It follows that Bill is also a knave.

Since Bill is a knave, it follows that Mel's statement that "Bill is a knave" is true, and hence that Mel is a knight.

Since Bob claims that neither Mel nor Bill are knaves, and Bill is a knave, Bob's statement is false, so Bob is a knave, too.

Since Bob is a knave, Marge's statement that Bob is not a knave is false, so Marge is a knave.

Since Marge is a knave, Alice's claim that Zeke and Marge are both knights is false, and so Alice is also a knave.

Since Carl is a knave, Carl's statement that "I know that Alice is a knave and that Zeke is a knight" is false. As Alice is a knave, the statement can only fail if Zeke is not a knight, and so Zeke is a knave. (Notice that since Carl is a knave, Zeke's statement "Of Carl and I, exactly one is a knight" would be true if and only if Zeke is a knight, which is really kind of redundant by the definitions of knaves and knights.)

Thus Mel is a knight, but Marge, Betty, Bob, Bill, Carl, Zeke, and Alice are knaves. ■