Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2019

Assignment #8
Rational Thoughts
Due on Monday, 4 November.

Recall that we defined the rational numbers as follows:

The equivalence relation $\approx$ on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\}) = \{ (a,b) \mid a,\, b \in \mathbb{Z} \wedge b \neq 0 \}$ is defined by $(a,b) \approx (c,d) \iff ad = bc$. Then $\mathbb{Q} = \left\{ [(a,b)]_\approx \mid a,\, b \in \mathbb{Z} \wedge b \neq 0 \right\}$.

Intuitively, $(a,b) \approx (c,d)$ if the pairs have the same ratio, *i.e.* $\frac{a}{b} = \frac{c}{d}$, so the equivalence class $[(a,b)]_\approx$ represents the ratio $\frac{a}{b}$.

We then defined multiplication on the rational numbers by $[(a,b)]_\approx \cdot [(c,d)]_\approx = [(ac,bd)]_\approx$ and showed that it was commutative. Also, we let $0 = 0_\mathbb{Q} = [(0,1)]_\approx$ and $1 = 1_\mathbb{Q} = [(1,1)]_\approx$.

In what follows, you may assume that the usual algebraic operations on the integers have all the usual properties.

1. Show that non-zero rational numbers have multiplicative inverses: if $p \in \mathbb{Q}$ and $p \neq 0$, then there is a $p^{-1} \in \mathbb{Q}$ such that $pp^{-1} = 1$. *[5]*

2. Show that multiplication on $\mathbb{Q}$ satisfies the Cancellation Law: if p, q, $r \in \mathbb{Q}$, where $r \neq 0$, and $pr = qr$, then $p = q$. *[5]*

Recall that we're changing the due dates for our weekly assignments to Mondays instead of Fridays so that we can make more effective use of our seminars.