Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2019

Assignment #6
This and that
Due on Friday, 18 October.

Recall that if a and b are natural numbers, then $a \mid b$ means that a is a factor of b, *i.e.* $b = ac$ for some natural number c, and that a natural number $p > 1$ is said to be *prime* if it has no natural number factor other than itself and 1.

1. Suppose p is prime, a, $b \in \mathbb{N}$, and $p \mid ab$. Show that $p \mid a$ or $p \mid b$. *[3]*

2. Show that there are infinitely many prime numbers. *[2]*

Recall from class that we defined the set of integers by defining the equivalence relation $\sim$ on $\mathbb{N} \times \mathbb{N}$ by $(a, b) \sim (c, d) \implies a + d = c + b$, and then took the integers to be equivalence classes for this relation, *i.e.* $\mathbb{Z} = \left\{ [(a, b)]_\sim \mid (a, b) \in \mathbb{N} \times \mathbb{N} \right\}$. We then proceeded to define $0_\mathbb{Z} = [(0, 0)]_\sim$, $1_\mathbb{Z} = [(1, 0)]_\sim$, $-[(a, b)]_\sim = [(b, a)]_\sim$, $[(a, b)]_\sim + [(c, d)]_\sim = [(a + c, b + d)]_\sim$, and $[(a, b)]_\sim \cdot [(c, d)]_\sim = [(ac + bd, ad + bc)]_\sim$.

3. Show that $+$ is an asociative operation on $\mathbb{Z}$. *[2]*

4. Show that $\cdot$ is a well-defined operation on $\mathbb{Z}$. *[3]*