Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2019

Assignment #4 Make-Up
Sets of Threes
Due on Monday, 28 October.

This assignment is a make-up for Assignment #4, on which a number of students missed at least part of the point of the question. If you do this assignment, and do better than you did on Assignment #4, this one will count instead of Assignment #4 in computing your mark.

Here is a formal definition of a minimal first-order language for set theory:

The symbols of the language are as follows:

Variables: x_0, x_1, x_2, ...
Connectives: $\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$
Quantifiers: $\forall$, $\exists$
Parentheses: (,)
Equality: $=$
Set Membership: $\in$ (a 2-place relation)
Just to be paranoid: all of the above symbols are distinct, none is a substring of any other, and there are no other symbols in the language.

The formulas (*i.e.* statements) of the language are defined as follows:

1. For any variables x_i and x_j of the language, $(x_i = x_j)$ and $(x_i \in x_j)$ are formulas of the language.
2. If φ and ψ are any formulas of the language, then $(\neg\varphi)$, $(\varphi \vee \psi)$, $(\varphi \wedge \psi)$, $(\varphi \rightarrow \psi)$, and $(\varphi \leftrightarrow \psi)$ are also formulas of the language.
3. If φ is any formula of the language and x_i is any variable of the language, then $(\forall x_i \varphi)$ and $(\exists x_i \varphi)$ are also formulas of the language.
4. No string of symbols of the language is a formula of the language unless it was formed using (possibly many applications of) rules 1–3 above.

This language is inefficient in some ways – it could really use a symbol for the empty set and some additional relations, such as the subset relation, and overuses parentheses, among other things – but as first-order languages go it is pretty uncomplicated. To compensate for its deficiencies, one usually augments this language informally with auxiliary symbols for common objects (*e.g.* $\emptyset$), operations (*e.g.* $\cup$, $\cap$, $\setminus$), and relations (*e.g.* $\subseteq$, $\subsetneqq$), as well as *ad hoc* names for generic sets (*e.g.* A, B).

Informally, define a "set of threes" to be a set in which every element is a set of three elements.

1. Define "set of threes" in the (formal and unaugmented) given language. *[10]*

NOTE: This means that you need to write a formula in the language which is true exactly when a variable happens to be a "set of threes". Of course, that variable had better appear in the formula.