

## Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2019

### Assignment #4

#### Ordered Pairs

Due on Friday, 4 October.

Here is a formal definition of a minimal first-order language for set theory:

The symbols of the language are as follows:

*Variables:*  $x_0, x_1, x_2, \dots$

*Connectives:*  $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$

*Quantifiers:*  $\forall, \exists$

*Parentheses:*  $(, )$

*Equality:*  $=$

*Set Membership:*  $\in$  (a 2-place relation)

Just to be paranoid: all of the above symbols are distinct, none is a substring of any other, and there are no other symbols in the language.

The formulas (*i.e.* statements) of the language are defined as follows:

1. For any variables  $x_i$  and  $x_j$  of the language,  $(x_i = x_j)$  and  $(x_i \in x_j)$  are formulas of the language.
2. If  $\varphi$  and  $\psi$  are any formulas of the language, then  $(\neg\varphi)$ ,  $(\varphi \vee \psi)$ ,  $(\varphi \wedge \psi)$ ,  $(\varphi \rightarrow \psi)$ , and  $(\varphi \leftrightarrow \psi)$  are also formulas of the language.
3. If  $\varphi$  is any formula of the language and  $x_i$  is any variable of the language, then  $(\forall x_i \varphi)$  and  $(\exists x_i \varphi)$  are also formulas of the language.
4. No string of symbols of the language is a formula of the language unless it was formed using (possibly many applications of) rules 1–3 above.

This language is inefficient in some ways – it could really use a symbol for the empty set and some additional relations, such as the subset relation, and overuses parentheses, among other things – but as first-order languages go it is pretty uncomplicated. To compensate for its deficiencies, one usually augments this language informally with auxiliary symbols for common objects (*e.g.*  $\emptyset$ ), operations (*e.g.*  $\cup, \cap, \setminus$ ), and relations (*e.g.*  $\subseteq, \subsetneq$ ), as well as *ad hoc* names for generic sets (*e.g.*  $A, B$ ).

1. Define “ordered pair” in the (formal and unaugmented) given language. [10]

NOTE: The ordered pair  $(a, b)$  is different from the ordered pair  $(b, a)$  unless  $a = b$ . Your first problem for **1** is to figure out what it actually means to define such a concept in the given language.