

**Mathematics 2200H – Mathematical Reasoning**

TRENT UNIVERSITY, Fall 2019

**Assignment #11**

**Real Cardinality**

*Due on Monday, 25 November.*

Recall that two sets  $A$  and  $B$  have equal cardinality, written as  $|A| = |B|$ , if there is a 1–1 onto function  $f : A \rightarrow B$ . If all we have is that there is a 1–1 function  $g : A \rightarrow B$ , then  $|A| \leq |B|$ ; the Schröder-Bernstein Theorem, which we will do in class soon, tells us that  $|A| \leq |B|$  and  $|B| \leq |A|$  together imply that  $|A| = |B|$ .

1. Without using the Schröder-Bernstein Theorem, show that  $|\mathbb{R}| = |\mathbb{R} \times \mathbb{R}|$ . [10]

NOTE. You may assume that the real numbers and the basic operations on them have been defined and have the usual properties. Just in case, recall that  $\mathbb{R} \times \mathbb{R} = \{ (a, b) \mid a, b \in \mathbb{R} \}$ .