Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2017

Solutions to Assignment #3
Knights and Knaves

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet seven inhabitants: Joe, Sue, Sally, Bozo, Dave, Zed and Alice. Joe says that Dave could claim that Alice is a knave. Sue says that Bozo and Joe are both knights or both knaves. Sally claims, "At least one of the following is true: that I am a knight or that Sue is a knave." Bozo claims, "Sally is a knave." Dave claims that Alice is a knight or Bozo is a knight. Zed claims, "Of I and Sally, exactly one is a knight." Alice says, "Dave could say that Zed is a knave."

1. Determine, as best you can, which of the seven are knights and which are knaves. *[10]*

SOLUTION. There are many ways to reason this out – I came up with four different ways while trying to find the neatest way I could – and any correct and complete method is fine. Here's my favourite among the methods I came up with:

> First, consider Zed's statement. If Zed is a knight, this statement must be true, in which case Sally must be a knave. On the other hand, if Zed is a knave, the statement must be false, and the only way for that to be so in this case is for Sally to be a knave. Thus Sally must be a knave no matter what.
>
> Second, since Sally is a knave, Bozo's statement is true, so Bozo must be a knight.
>
> Third, again since Sally is a knave, Sally's own statement is false, which, in particular, means that Sue cannot be a knave. Thus Sue must be a knight.
>
> Fourth, since Sue is a knight, her statement must be true, and because we already know that Bozo is a knight, it follows that Joe is a knight, too.
>
> Fifth, since Bozo is a knight, Dave's statement (interpreting "or" inclusively, as is our default) is true, so Dave must be a knight also.
>
> Sixth, since Joe is a knight and Dave is a knight, Joe's statement about what Dave could claim can only be true if Alice is a knave.
>
> Seventh, since Alice is a knave and dave is a knight, Alice's statement about what Dave could say can only be false if Zed is a knight.
>
> Thus Bozo, Dave, Joe, Sue, and Zed are knights, while Alice and Sally are knaves.

Given the conclusion about each person's status, one should check, just in case, that each statement is properly true or false according to the status of the person making it. I'll leave that to you, if you think it necessary. □

NOTE. The problem given above is Puzzle #273 out of 382 from the collection at:

`philosophy.hku.hk/think/logic/knights.php`