

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2017

Assignment #9

Some counting

Due on Thursday, 16 November.

1. Show that if $n \geq 1$, then $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0$. [2]
2. What does $\sum_{k=0}^n \frac{1}{k!(n-k)!}$ add up to? Why? [2]
3. Without using the fact that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, show that $\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^2 = \sum_{n=0}^{\infty} \frac{2^n}{n!}$. [2]
4. Let ${}^{\mathbb{N}}2 = \{f \mid f : \mathbb{N} \rightarrow \{0, 1\}\}$ be the set of all functions from \mathbb{N} to $2 = \{0, 1\}$, and let $\mathcal{P}(\mathbb{N}) = \{X \mid X \subseteq \mathbb{N}\}$ be the set of all subsets of \mathbb{N} . Show that $|{}^{\mathbb{N}}2| = |\mathcal{P}(\mathbb{N})|$. [4]