Assignment #4
Induction
Due on Thursday, 5 October.

Recall that $n! = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$.

1. Use induction to show that $\dfrac{n^n}{3^n} < n! < \dfrac{n^n}{2^n}$ for all $n \geq 6$. *[5]*

NOTE. You could think of this result as a first, crude, cut at Stirling's Formula, which lets one approximate $n!$ for large n.

2. Without looking it up, try to find or guess a formula for the sum of the first n cubes,

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^2 + 3^3 + \cdots + (n-1)^3 + n^3 \,,$$

and then use induction to verify that your formula is true. *[5]*

Hint: This has something a little surprising to do with the fact that

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2} \,.$$