**Mathematics 2200H – Mathematical Reasoning**
TRENT UNIVERSITY, Fall 2017

**Assignment #2**
**Digits and Divisibility**
*Due on Thursday, 21 September.*

Suppose $b \in \mathbb{N}$ and $b > 1$. If $n$ is any positive natural number, then there are unique natural numbers $k$ and $d_k$, $d_{k-1}$, ... , $d_0$, such that $0 \leq d_i < b$ for each $i$ with $0 \leq i \leq k$, $d_k \neq 0$, and $n = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_0 b^0$. The base $b$ representation of $n$ is then the sequences of digits $d_k d_{k-1} \ldots d_0$; if we need to emphasize that it is in base $b$, we will often write something like $(d_k d_{k-1} \ldots d_0)_b$. For example, consider $533 = (533)_{10}$. 533 would be written as 1000010101 in base 2 because $533 = 512 + 16 + 4 + 1 = 1 \cdot 2^9 + 0 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$.

**1.** Suppose $n = (d_k d_{k-1} \ldots d_1 d_0)_{10}$. Show that $3 \mid n \iff 3 \mid (d_k + d_{k-1} + \cdots + d_1 + d_0)$. For which natural numbers $> 1$ other than 3 does this trick work? *[5]*

**2.** Given a base $b$, for which integers $c$ is it true that whenever $n = (d_k d_{k-1} \ldots d_1 d_0)_b$, we have $c \mid n \iff c \mid (d_k + d_{k-1} + \cdots + d_1 + d_0)$? *[2]*

**3.** Devise a similar trick for testing for divisiblity by 11 in base 10. *[3]*

**On Problems**

Problems worthy
    of attack
Prove their worth
    by hitting back.

*Another grook by Piet Hein.*