

**Mathematics 2200H – Mathematical Reasoning**

TRENT UNIVERSITY, Fall 2017

**Assignment #10**

**More counting**

*Due on Thursday, 23 November.*

A set  $A$  is said to be *countable* if  $|A| \leq |\mathbb{N}|$ , and *countably infinite* if  $|A| = |\mathbb{N}|$ .

1. Suppose  $A_n$ ,  $n \in \mathbb{N}$ , is a countably infinite collection of disjoint countably infinite sets. (So each  $A_n$  is countably infinite and  $A_m \cap A_k = \emptyset$  whenever  $k \neq m$ .) Show that  $A = \bigcup_{n=0}^{\infty} A_n$  is also countably infinite. [4]
2. Suppose  $C$  is an infinite subset of a countably infinite set  $D$ . Show that  $C$  is also countably infinite.
3. Suppose  $A$  is countable and there is an onto function  $F : A \rightarrow B$ . Show that  $B$  is countable. [3]