Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2016

Solutions to Assignment #5
Rock, paper, scissors expanded.

In the humorous Harry Potter fanfiction story *Inspected by No 13*[†], by Clell65619[‡], the third task of the Triwizard Tournament is, due to budget problems, replaced by a game which is an extension of Rock, Paper, Scissors:

> "Well, all our champions are ready for the third task, which will start on my command. Due to certain ... unexpected difficulties, the original third task had to be reconsidered. At one point it was going to be a race through a magical maze, but instead, to decide our ultimate Triwizard champion we are going to conduct we are going to conduct that most challenging of tasks ... A round of Parchment, Stone, Wand, Dragon, Merlin."
>
> Harry blinked. Seriously? In his time at Hogwarts he had witnessed dozens of games of Parchment, Stone, Wand, Dragon, Merlin, which always took hours because the Purebloods always picked the same thing for several rounds until it occurred them that it wasn't working.
>
> "For the benefit of the Muggle raised in the audience," Bagman continued, "Playing the game is perfectly simple. Wand cuts Parchment," he explained pantomiming the actions. "Parchment covers stone. Stone crushes dragon. Dragon eats Merlin. Merlin snaps Wand. Wand stuns dragon. Dragon burns Parchment. Parchment confuses Merlin. Merlin vanishes stone. And, of course, stone breaks Wand."
>
> "What was that again?" Harry asked, wondering if he could get the fraud to repeat himself.
>
> "Of course, It's very simple. Wand cuts Parchment," he explained, again pantomiming the actions. "Parchment covers stone. Stone crushes dragon. Dragon eats Merlin. Merlin snaps Wand. Wand stuns dragon. Dragon burns Parchment. Parchment confuses Merlin. Merlin vanishes stone. And, of course, stone breaks Wand."
>
> "Ok," Harry nodded. "Got it."
>
> Bagman raised his wand and a white square appeared in the middle of the pitch. Harry took his assigned place as the others took there.
>
> "The Winner of this Third Task wins the Tournament over all. Are all the Champions ready?" Bagman announced.
>
> All four of the champions signaled their readiness. Harry looked at the other three players who had become his friends over the last few months. They were all purebloods, even Fleur after a fashion.
>
> Would they all be as predictable as the purebloods at Hogwarts?

[†] Available on FanFiction.net at `https://www.fanfiction.net/s/10485934/1/` .

[‡] His page on FanFiction.net is at `https://www.fanfiction.net/u/1298529/Clell65619` - it has links to a number of other fanfiction stories of his for various fandoms, some of which are very good.

"This is it," Bagman continued. "This is what we've been waiting for, this is the big one. This is the one that ... "

"Would you just get on with it?" Barty Crouch demanded from the judges stands.

"Oh, of course. Certainly," Bagman said. "Champions ready? One Two Three Cast"

As one, four hands shot forward and Harry made his decision.

1. In the story, Harry casts Parchment, defeating the other champions, all of whom do cast Merlin. What should Harry have cast, just in case one of the others had the same idea? *[5]*

SOLUTION. Harry should have cast Dragon. If any other player had the idea of defeating Merlin by casting Parchment, well, Dragon beats both. Moreover, if another player had *this* idea and cast Dragon too, it's a tie and Harry would still be in the game for another round. ■

2. Can one create nice symmetric Rock, Paper, Scissors type games with four, six, or seven possible casts? If so, describe them; if not, explain why not? *[5]*

SOLUTION. You can't with four or six (or any even number of) casts because each cast would have to have a different number of casts defeating it as it defeats (since there would be an odd number of *other* casts), or else some casts would neither defeat nor be defeated by some others. Neither alternative would be in the spirit of Rock, Paper, Scissors.

One can do this with seven (or any odd number of) casts, precisely because there is an even number of *other* casts for each cast, so it can defeat half and be defeated by half, preserving symmetry. For example, with seven casts, we could have $1 \to 2, 3, 4$, $2 \to 3, 4, 5$, $3 \to 4, 5, 6$, $4 \to 5, 6, 7$, $5 \to 6, 7, 1$, $6 \to 7, 1, 2$, and $7 \to 1, 2, 3$, where $\to$ means "defeats". ■

Bonus. Notice that the hand signs for the casts in Parchment, Stone, Wand, Dragon, Merlin are never actually described. Make up descriptions for each of them. *[1]*

SOLUTION. I wouldn't ask this one if I had any good ideas! ■