

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2016

Solution to Assignment #2

Knights and Knaves

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet seven inhabitants: Mel, Joe, Zed, Bart, Marge, Bozo and Carl. Mel claims, “Bart is a knight and Bozo is a knave.” Joe claims that only a knave would say that Marge is a knave. Zed claims that at least one of the following is true: that Bart is a knight or that Bozo is a knave. Bart says that both Mel is a knight and Zed is a knave. Marge tells you, “Mel is a knave.” Bozo tells you that Carl is a knave. Carl says, “Bozo is a knave or Bart is a knave.”

Can you determine who is a knight and who is a knave?[†]

1. Determine, as best you can, which of the seven inhabitants is a Knight and which is a Knave. [10]

NOTE: Do make sure to provide as complete and coherent reasoning as you can as part of your answers to both problems above!

SOLUTION. (*Look, Ma! No symbols or truth tables!*) First, Carl cannot be a knave. Suppose otherwise. If Carl is a knave, then his statement “Bozo is a knave or Bart is a knave” must be false. Since the statement would be true if at least one of Bozo or Bart were actually a knave, it can only be false if both Bozo and Bart are knights. It follows that Bart’s assertion that Mel is a knight and Zed is a knave must be true, so, in particular, Mel must be a knight. It follows in turn that Mel’s claim that “Bart is a knight and Bozo is a knave” must be true, so, in particular, Bozo must be a knave. However this contradicts the fact that Bozo must be a knight if Carl is a knave. Thus, Carl cannot be a knave, and so must be a knight.

Second, since Carl is a knight, Bozo’s statement that Carl is a knave is false, and so Bozo must be a knave. Since Bozo is a knave, Zed’s claim that at least one of “Bart is a knight or that Bozo is a knave” is true is correct, and so Zed is a knight. Since Zed is not a knave, Bart’s statement that Mel is a knight and Zed is a knave is false, so Bart is knave. Since Bart is not a knight, Mel’s claim that “Bart is a knight and Bozo is a knave” is false, and so Mel is also a knave. Since Mel is indeed a knave, Marge’s statement that “Mel is a knave” is true, so Marge is a knight. This makes Joe’s assertion that only a knave would say that Marge is a knave correct, and so Joe is also a knight.

Bottom line: there is only one solution, namely that Carl, Joe, Marge, and Zed are knights, and that Bart, Bozo, and Mel are knaves. ■

[†] This is puzzle #270 out of 382 from a collection of knights and knaves puzzles at:
philosophy.hku.hk/think/logic/knights.php