

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2016

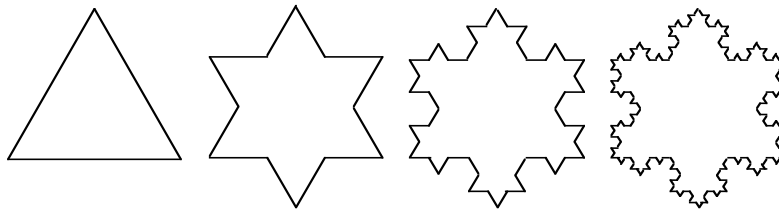
Assignment #9

A flaky limit or two

Due on Thursday, 17 November.

Rent University decided to build a new parking lot, to be completely enclosed by a fence on its perimeter, whose shape reflected Rent's slightly flaky reputation. The relevant University committee decided that it would be cool to build a fence in the shape of the *snowflake curve* (also known as the *Koch curve*), which is defined as follows.

Start with an equilateral triangle. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing "tooth," both of whose sides are as long as the removed third, one gets a six-pointed star. Suppose one repeats this process for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on, as in the diagram:



Note that the lengths of the line segments at each stage are a third of the length of the segments at the preceding stage. The curve which is the limit of this process (after infinitely many steps) is the snowflake curve.

Rent University hired your employer, Wreckit and Offspring, to pave the parking lot and build the perimeter fence. Your employer, in turn, handed you the following tasks related to this contract. You may assume that the equilateral triangle at the start of the definition of the snowflake curve has sides one *fimflam* long. (This special unit of length was invented by Wreckit and Offspring just for this contract: Rent University insisted on the project being as unique as possible. :-)

1. Your first task is to determine the length of the fence. [5]
2. Your second task is to determine the area of the parking lot. [5]

NOTE: The snowflake curve is an example of a curve which is continuous, but not differentiable. It fails to be differentiable thanks to the infinitely many densely-packed corners it has.