

Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2016

Assignment #4

A little formality

Due on Thursday, 6 October.

Here is a formal definition of a fairly minimal first-order language for set theory:

The symbols of the language are as follows:

Variables: x_0, x_1, x_2, \dots

Connectives: $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$

Quantifiers: \forall, \exists

Parentheses: $(,)$

Equality: $=$

Set Membership: \in (a 2-place relation)

All of the above symbols are distinct, none is a substring of any other, and there are no other symbols in the language.

The formulas (*i.e.* statements) of the language are defined as follows:

1. For any variables x_i and x_j of the language, $(x_i = x_j)$ and $(x_i \in x_j)$ are formulas of the language.
2. If φ and ψ are any formulas of the language, then $(\neg\varphi)$, $(\varphi \vee \psi)$, $(\varphi \wedge \psi)$, $(\varphi \rightarrow \psi)$, and $(\varphi \leftrightarrow \psi)$ are also formulas of the language.
3. If φ is any formula of the language and x_i is any variable of the language, then $(\forall x_i \varphi)$ and $(\exists x_i \varphi)$ are also formulas of the language.
4. No string of symbols of the language is a formula of the language unless it was formed using (possibly many applications of) rules 1–3 above.

This language is inefficient in some ways – it could really use a symbol for the empty set and some additional relations, such as the subset relation, and overuses parentheses, among other things – but as first-order languages go it is pretty uncomplicated.

1. What are the possible lengths of formulas of the given language? [5]

NOTE: The length of a formula of the language is the number of symbols of the language making up the formula, counting repetitions. For example, each instance of a variable x_i counts as one symbol.

2. Find a way to define ordered pairs in the given language. [5]

NOTE: The ordered pair (a, b) is different from the ordered pair (b, a) unless $a = b$. Your first problem for **2** is to figure out what it actually means to define such a concept in the given language.