Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2016

Assignment #10
Prime Time Counting
Due on Thursday, 24 November.

Recall that a natural number $p > 1$ is said to be *prime* if it has no positive integer factors other than itself and 1; otherwise it is said to be *composite*.

1. Use the familiar algebraic properties of $+$ and $\cdot$ on $\mathbb{N}$ to help show that every positive integer $n > 1$ has a prime factor. *[5]*

HINT: This could be done by induction, or "complete" induction, or reverse induction, or the least number principle, or ...

An infinite set A is said to be *countable* if it can be listed, *i.e.* $A = \{ a_0, a_1, a_2, \dots \}$. Technically, this means that there is a 1-1 onto function $f : \mathbb{N} \to A$, the idea being that $f(n) = a_n$ gives you the nth element of the list.

2. Show that the set of natural numbers, $\mathbb{N}$, is itself countable. *[1]*

HINT: A *really* simple listing does the job.

3. Show that the set of integers, $\mathbb{Z}$, is countable. *[1]*

HINT: Even ns list non-negative integers, odd ns list negative integers.

4. Show that the set of rational numbers, $\mathbb{Q}$, is countable. *[3]*

HINT: Read. Look it up. It's a neat trick!

NOTE: It turns out that not every infinite set is countable. In particular, as we'll see in class in a while, the set of real numbers, $\mathbb{R}$, is *not* countable.