

**Mathematics 2200H – Mathematical Reasoning**

TRENT UNIVERSITY, Fall 2015

**Assignment #9**

*Due on Monday, 23 November, 2015.*

**Epsolinics Epsilinoes Epsilonics**

Recall that a *sequence* of real numbers is a list indexed by the natural numbers,

$$a_0, a_1, a_2, a_3, \dots, a_n, \dots$$

usually denoted by something like  $\{a_n\}$ . The *limit* of a sequence  $\{a_n\}$  is a real number  $L$ , usually denoted by  $\lim_{n \rightarrow \infty} a_n = L$ , if

for every real number  $\varepsilon > 0$ , there is some  $N \in \mathbb{N}$ ,  
such that for every  $n \geq N$ ,  $|a_n - L| < \varepsilon$ ,

*i.e.*  $\forall \varepsilon > 0 \exists N \in \mathbb{N} \forall n \in \mathbb{N} : n \geq N \implies |a_n - L| < \varepsilon$ .

1. Suppose  $\{a_n\}$  is a sequence such that  $\lim_{n \rightarrow \infty} a_n = L \neq 0$ . Show that if  $b_n = \frac{1}{a_n}$ , then

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{a_n} = \frac{1}{L}. \quad [10]$$