Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2015

Assignment #8
Due on Monday, 16 November, 2015.

Suprema and Infima

Recall that the *supremum* or *least upper bound* of a non-empty set X of real numbers is the real number s such that:

- s is an upper bound for X, *i.e.* $x \leq s$ for all $x \in X$.
- If u is any upper bound for X, then $s \leq u$.

Similarly, the *infimum* or *greatest lower bound* of a non-empty set X of real numbers is the real number i such that:

- i is a lower bound for X, *i.e.* $i \leq x$ for all $x \in X$.
- If ℓ is any lower bound for X, then $\ell \leq i$.

Note that X can have a supremum, usually denoted by $\sup(X)$, only if it is bounded above, and an infimum, usually denoted by $\inf(X)$, only if it is bounded below.

Recall also that in terms of the definition of real numbers as schnitts (of rational numbers), $\sup(X) = \bigcup_{x \in X} x$ whenever X is a set of real numbers bounded above.

1. Using the definition of real numbers as schnitts, show that $\inf(X) = \bigcap_{x \in X} x$ whenever X is a set of real numbers bounded below. *[3]*

2. Suppose $\emptyset \neq X \subset \mathbb{R}$ is bounded above, and let $Y = \{\, -x \mid x \in X \,\}$. Show that Y is bounded below and that $\inf(Y) = -\sup(X)$. *[3]*

3. Suppose $\emptyset \neq Y \subseteq X \subset \mathbb{R}$ and X (and hence also Y) is bounded above and below. Show that $\inf(X) \leq \inf(Y) \leq \sup(Y) \leq \sup(X)$. *[3]*

4. Suppose $\emptyset \neq X \subset \mathbb{R}$ is a set such that $\inf(X) = \sup(X)$. What can you deduce about X? *[1]*