Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2015

Assignment #6

Due on Monday, 2 November, 2015.

An itty-bitty teeny-weeny polka dot infinity ...

An infinite set A is said to be *countable* if it can be listed, *i.e.* $A = \{ a_0, a_1, a_2, \dots \}$. Technically, this means that there is a 1-1 onto function $f : \mathbb{N} \to A$, the idea being that $f(n) = a_n$ gives you the nth element of the list.

1. Show that $\mathbb{N}$ itself is countable. *[1]*

HINT: A really simple listing does the job.

2. Show that $\mathbb{Z}$ is countable. *[2]*

HINT: Even ns list non-negative integers, odd ns list negative integers.

3. Show that $\mathbb{Q}$ is countable. *[4]*

HINT: Read. Look it up. It's a neat trick!

4. Show that $\mathbb{R}$ is *not* countable. *[3]*

HINT: Assume it was countable. List the decimal expansions of all the real numbers and use the list to make a decimal expansion that is not on the list.