Mathematics 2200H – Mathematical Reasoning

TRENT UNIVERSITY, Fall 2015

Assignment #5

Due on Monday, 19 October, 2015.

A prime little infinitude

Recall from somewhere [You musthave seen this, darn it!] that an integer $p > 1$ is said to be *prime* if it has no positive integer factors other than itself and 1; otherwise it is said to be *composite*.

1. Use the familiar algebraic properties of $+$ and $\cdot$ on $\mathbb{N}$ to help show that every positive integer $n > 1$ has a prime factor. *[5]*

HINT: This could be done by induction, or "complete" induction, or reverse induction, or the least number principle, or ...

2. Use the familiar algebraic properties of $+$ and $\cdot$ on $\mathbb{N}$, as well as the result in question 1, to help show that there are infinitely many prime numbers. *[5]*

HINT: Argue by contradiction that there cannot be only finitely many prime numbers.